Minimal Model of Majoronic Dark Radiation and Dark Matter and its Phenomenology

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- Introduction
- The Model
- Numerical study
- Phenomenology

- BSM New phys. are called for: (1) $\Omega_{DM} h^2 \sim 0.12$ (2) and $m_\nu \neq 0$
- DM: something BSM electrically charge neutral and stable/long-lived.
- (Too) Many models for Majorana ν . The key is the effective Weinberg operator $(LH)^2$ which breaks $U(1)_L$.
- Accidental global $U(1)_L \in SM$ and it connects to m_{ν} .
- Majorona mass is controlled by the scale of U(1)_L SSB in the type-I (and inverse see-saw) (PLB730, 347.)

$$y\overline{N}^{c}NS_{L} \rightarrow m_{N} = y\langle S_{L} \rangle$$

- DM is stabilized by the Krauss-Wilczek, $U(1)_L \rightarrow Z_2$.
- Global SSB $U(1)_L$ DM- m_{ν} model: Goldstone is built in. It contributes to radiation energy density.

Need New Physics-2:effective neutrino number

• Neutrinos decouple at $T \sim 1$ MeV ($\Gamma_{weak} \leq H$). Later, the photons were heated up by e^+e^- annihilation,

$$T_{
u} = \left(rac{4}{11}
ight)^{1/3} T_{\gamma} \sim 2K$$

• The present relativistic energy density of the universe

$$\rho_{rad} = g_{\gamma} \frac{\pi^2}{30} T_{\gamma}^4 + g_{\nu} \frac{\pi^2}{30} \frac{7}{8} T_{\nu}^4 = \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_{eff} \right] \rho_{\gamma}$$

- Taking into account the incomplete decoupling, $N_{eff}^{SM} = 3.046$ (Mangano et al. 2005). Nonzero $\triangle N_{eff} = N_{eff} 3.046$ call for new relativistic DOF beyond the SM.
- This new DOF is coined as dark radiation.

N_{eff} After Planck 2015

Planck 2015, 1502.01589, N_{eff} = 3.15(46) at 95%CL.
 Although the SM seems OK, the statistical significance to rule out DR is still very poor.



- $2.5 < N_{eff} < 3.5$ at 95%CL, Valentino et el, PRD93,083523.
- $\triangle N_{eff} = 0.29(15)$, G. Steigman, June 2015, INT talk.
- $\triangle N_{eff} = 0.4 1.0$, Riess et el(WFC3 on HST), Astrophys.J. 826 (2016)

One more thing to be taken into account

$$(\overline{\nu^{c}},\overline{\nu_{R}})\begin{pmatrix} 0 & y_{D}v_{SM} \\ y_{D}v_{SM} & M_{N}(=y_{s}v_{l}) \end{pmatrix} \begin{pmatrix} \nu \\ \nu_{R}^{c} \end{pmatrix}$$

•
$$\mu_{VS}^{SM}\simeq 10^{10-12}~{
m GeV}$$

A. Djouadi / Physics Reports 457 (2008) 1-216



• For ϕ_A, ϕ_B in $V = \lambda_A \phi_A^4 + \lambda_B \phi_B^4 + \lambda_{AB} \phi_A^2 \phi_B^2 + ..., \lambda_A > 0$, $\lambda_B > 0, \lambda_{AB} > -2\sqrt{\lambda_A \lambda_B}$ at any given energy scale. RGE study is necessary.

- Motivation and experimental facts
- The Model
- Numerical study
- Phenomenology

Model

Particle content:

	L, Z_2	<i>SU</i> (2)	$U(1)_Y$
S	2 _{SSB,+}	1	0
Φ	1_{-} (DM candidate)	1	0
Н	0 _{SSB,+}	2	$\frac{1}{2}$
N _{iR}	1_	1	0
Li	1_	2	$-\frac{1}{2}$

Renormalizable Lagrangian: (8 new parameters)

$$\mathcal{L}_{scalar} = (D_{\mu}H)^{\dagger}(D^{\mu}H) + (\partial_{\mu}\Phi)^{\dagger}(\partial^{\mu}\Phi) + (\partial_{\mu}S)^{\dagger}(\partial^{\mu}S) - V(H, S, \Phi)$$
$$V(H, S, \Phi) = -\mu^{2}H^{\dagger}H - \mu_{s}^{2}S^{\dagger}S + m_{\Phi}^{2}\Phi^{\dagger}\Phi + \lambda_{H}(H^{\dagger}H)^{2} + \lambda_{\Phi}(\Phi^{\dagger}\Phi)^{2} + \lambda_{s}(S^{\dagger}S)^{2} + \lambda_{SH}(S^{\dagger}S)(H^{\dagger}H) + \lambda_{\Phi H}(\Phi^{\dagger}\Phi)(H^{\dagger}H) + \lambda_{\Phi S}(S^{\dagger}S)(\Phi^{\dagger}\Phi) + \frac{\kappa}{\sqrt{2}} \left[(\Phi^{\dagger})^{2}S + S^{\dagger}\Phi^{2} \right]$$

and we take κ to be real, $m_{\Phi}^2 > 0$, and define $\bar{\kappa} \equiv \lambda_{\Phi S} v_s + \kappa$.

Model

After SSB, (S) ≠ 0 and (H) ≠ 0, the fields are expanded as Φ = 1/√2(ρ + iχ), S = 1/√2(v_s + s + iω) and for the Higgs H = (0, v+h/√2)^T. ω is the massless Goldstone or Majoron.
(S) is inv. under a U(1)_L π-rotation, a Z₂ parity remains:

 $s, \omega, h \longrightarrow s, \omega, h$ $\rho, \chi \longrightarrow -\rho, -\chi$

• Due to mixing, the physical mass eigenstates are

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h \\ s \end{pmatrix}$$

with mixing angle $\tan 2\theta = \frac{\lambda_{HS}v_Hv_S}{\lambda_Sv_S^2 - \lambda v_H^2}$.

• Identify $h_1 \equiv h_{SM}$ with a mass of 125 GeV, h_2 is a new neutral scalar(just call them H and S).

Model

• In terms of the physical masses and mixing, we have

$$\begin{split} \lambda_H &= \frac{\cos^2 \theta M_H^2 + \sin \theta^2 M_S^2}{2v_H^2}, v_S = -\frac{\sin \theta \cos \theta (M_H^2 - m_S^2)}{v_H \lambda_{SH}}, \\ \lambda_S &= \frac{\sin^2 \theta M_H^2 + \cos \theta^2 M_S^2}{2v_S^2}, y_S = \frac{\sqrt{2}M_N}{v_S}. \end{split}$$

•
$$v_H = 246 \text{GeV}, \ m_H = 125 \ \text{GeV}$$

- for a given set of $\{M_S, \theta, \lambda_{SH}\}$, v_S and λ_S are determined.
- No solution found for $M_N < 0.5 \text{TeV}$, not sensitive otherwise. We take $M_N = 1 \text{TeV}$ as a benchmark value.
- leptons interact with the Majoron via

$$\frac{1}{2v_s}(\partial_\mu\omega\,\bar\psi_I\gamma^\mu\psi_I)$$

 Very small(∝ m_ν) pseudoscalar couplings to u, d, e at 1-loop, no constraints from stellar cooling

T_{dec} of Majoron

• However a dim-5 int.

$$\mathcal{L}_{f\omega} = -rac{\lambda_{HS}m_f}{M_h^2 M_s^2} ar{f} f \partial^\mu \omega \partial_\mu \omega$$

can be generated through scalar mixing:



• Order of magnitude estimation gives

$$\Gamma(f\bar{f}\leftrightarrow\omega\omega)\simrac{\lambda_{HS}^2m_f^2}{M_H^4M_S^4} imes T_{
m dec}^7 imes N_c^4$$

• Since $H \sim T_{dec}^2/M_{Pl}$, $\frac{N_c \lambda_{HS}^2 m_{eff}^2 T_{dec}^5 M_{Pl}}{M_H^4 M_s^4} \approx 1.$

• Conservation of Entropy in the co-moving volume give:

$$\triangle N_{eff} = \frac{4}{7} \left(\frac{g_*(T_\nu^+)}{g_*(T_\omega^-)} \right)^{\frac{4}{3}}$$

where g_* is the effective number of relativistic DOF. $\Delta N_{\rm eff} = \{0.39, 0.055, 0.0451, 0.0423\}$ for $T_{dec} = \{m_{\mu}, 1GeV, m_c, m_{\tau}\}$ respectively.

 Due to scalar mixing, H can always decays into a pair of invisible ω's,

$$\Gamma_{\omega\omega} = \frac{1}{32\pi} \frac{\sin^2 \theta M_H^3}{v_S^2}$$

• $\Gamma_{\omega\omega} \leq \Gamma_{H}^{inv} < 0.8$ MeV gives M_{S}^{max} via

$$\frac{M_{S}^{4}}{(M_{H}^{2}-M_{S}^{2})^{2}} \leq \cos^{2}\theta \frac{32\pi m_{eff}^{2} T_{dec}^{5} M_{pl}}{v_{H}^{2} M_{H}^{7}} \Gamma_{H}^{inv}$$

M_S , T_{dec} , and sin θ^2

• LHC-I, $\mu = 1.1 \pm 0.11$ gives indirect bound sin $\theta^2 < 0.13$ at 2 σ . Direct search from OPAL $e^+e^- \rightarrow hZ$.



- From rare B decay, $|\theta| < 0.002$ for $M_S < 2$ GeV.
- With this, the decoupling condition yields

$$\lambda_{SH} \sim rac{M_H^2 M_S^2}{T_{dec}^3 \sqrt{T_{dec} M_{pl}}} \ll 1$$

Relic density and direct detection



- Relic density/ SI scattering can be calculated.
- Thermal $\Omega_{DM}h^2 \sim \frac{0.1\text{pb}}{\langle \sigma v \rangle}$, $\langle \sigma v \rangle = 2.5(1) \times 10^{-9} (GeV)^{-2}$.
- limit from LUX $(M_{
 ho} \sim \mathcal{O}(TeV))$

1-loop beta function

$$\begin{split} 16\pi^2 \frac{d\lambda_H}{dt} &= 12\lambda_H^2 + 6\lambda_H y_t^2 - 3y_t^4 - \frac{3}{2}\lambda_H (3g_2^2 + g_1^2) + \frac{3}{16} \left[(g_1^2 + g_2^2)^2 + 2g_2^4 \right] \\ &\quad + \frac{1}{2} (\lambda_{HS}^2 + \lambda_{\Phi H}^2) \\ 16\pi^2 \frac{d\lambda_\Phi}{dt} &= 10\lambda_\Phi^2 + \lambda_{\Phi H}^2 + \frac{1}{2}\lambda_{\Phi S}^2 \\ 16\pi^2 \frac{d\lambda_S}{dt} &= 10\lambda_S^2 + \lambda_{HS}^2 + \frac{1}{2}\lambda_{\Phi S}^2 - 8Y_S^4 + 4Y_S^2\lambda_S \\ 16\pi^2 \frac{d\lambda_{HS}}{dt} &= 2\lambda_{HS}^2 + \lambda_{HS} (6\lambda_H + 4\lambda_S) + 2\lambda_{HS}Y_S^2 - \frac{3}{4}\lambda_{HS} (3g_2^2 + g_1^2) \\ &\quad + 3\lambda_{HS}y_t^2 + \lambda_{\Phi S}\lambda_{\Phi H} \\ 16\pi^2 \frac{d\lambda_{\Phi H}}{dt} &= 2\lambda_{\Phi H}^2 + \lambda_{\Phi H} (6\lambda_H + 4\lambda_{\Phi}) - \frac{3}{4}\lambda_{\Phi H} (3g_2^2 + g_1^2) \\ &\quad + 3\lambda_{\Phi H}y_t^2 + \lambda_{\Phi S}\lambda_{HS} \\ 16\pi^2 \frac{d\lambda_{\Phi S}}{dt} &= 2\lambda_{\Phi S}^2 + 4\lambda_{\Phi S} (\lambda_\Phi + \lambda_S) + 2\lambda_{\Phi S}Y_S^2 + 2\lambda_{\Phi H}\lambda_{HS} \\ 16\pi^2 \frac{dY_S}{dt} &= 3Y_S^3, \ 16\pi^2 \frac{d\kappa}{dt} = \kappa \left(2\lambda_{\Phi S} + 2\lambda_{\Phi} + Y_S^2 \right) \end{split}$$

- Fermionic y^4 contributions responsible for vacuum instability.
- In general, new scalar DOF's help VS.
- Possible new Landau pole in λ_{Φ}

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Numerical Scan

- Comprehensive scan of the whole parameter space. Randomly scan T_{dec} , M_S , θ , M_{ρ} , $\lambda_{\Phi S} (\in [-4\sqrt{\pi\lambda_S}, 4\pi])$, $\bar{\kappa}$, $\lambda_{\Phi H}$, λ_{Φ} .
- Requirements and experimental constraints in our search:
 - Improve the SM vacuum stability, $\mu_{VS} > \mu_{VS}^{SM}$ $(\mu_{VS1-loop}^{SM} = 2 \times 10^5 {\rm GeV})$
 - No Landau pole below μ_{VS}^{SM}
 - $\Gamma_{inv}^H < 0.8 \text{MeV}.$
 - $T_{dec} \in [m_{\mu}, 2GeV].$
 - $\bullet~\theta$ complies with all experimental bounds.
 - relic density $\langle \sigma v \rangle = 2.5(1) \times 10^{-9} (GeV)^{-2}$.
 - Spin-independent direct DM search bound (LUX)
- The largest $R_{VS} \equiv \log_{10} \mu_{VS} / \mu_{VS}^{SM}$ we got ~ 11 . New scalar DOF help to go up to GUT scale, but not M_{pl} .
- $T_{dec} > 1.3 {
 m GeV}$, $1.5 {
 m TeV} < M_{
 ho} < 4 {
 m TeV}$, $M_S \in [20, 100] {
 m GeV}$, $v_S, -\kappa \in [2-20] {
 m TeV}$

Some qualitative understanding

- $T_{dec} < 2 \text{GeV}$, small λ_{SH} .
- λ_S V.S. needs large v_S to suppress $y_S = \sqrt{2}M_N/v_s$.
- large $v_S \Rightarrow$ large mixing \Rightarrow large M_S and higher T_{dec} .
- To counter act y_S in λ_S V.S. \Rightarrow sizable $\lambda_{\Phi S}$.
- to improve λ_H V.S. $\Rightarrow \lambda_{\Phi H} \sim \mathcal{O}(1)$.
- $\lambda_{\Phi H} \sim \mathcal{O}(1) \Rightarrow M_{
 ho} > 1.5 \text{TeV}.$
- to avoid λ_{Φ} Landau pole \Rightarrow small $\lambda_{\Phi}, \lambda_{\Phi S}$ are preferred.
- small $\lambda_{\Phi S} \Rightarrow M_{\rho} < 4$ TeV, or too much DM.



 R_{VS} G:2 – 4, B:4 – 6, R:> 6.

Numerical Scan: 2 examples

Config.	T _{dec}	Ms	θ	$M_{ ho}$	VS	R _{VS}	$Br(\omega\omega)$	Br(bb)
A	1.94	27.3	-0.03	2.2	6.7	2.1	0.87	0.11
В	1.87	67.6	-0.32	1.8	12.1	10.0	0.07	0.78

 T_{dec} and M_S (M_{ρ} and v_S) are in GeV (TeV).



Numerical Scan:DM annihilation

For $\lambda_S, \lambda_{SH} \ll 1$ and $M_\rho \gg M_W, M_Z, M_H$, the total annihilation cross section of $\rho \rho \rightarrow W^+ W^-, ZZ, HH$ can be estimated to be

$$egin{aligned} \langle \sigma \mathbf{v}
angle_{W/Z/H} &\equiv \langle \sigma \mathbf{v}
angle_{W^+W^-} + \langle \sigma \mathbf{v}
angle_{ZZ} + \langle \sigma \mathbf{v}
angle_{HH} &\sim rac{1}{64\pi} rac{\lambda_{\Phi H}^2}{M_
ho^2} imes [2+1+1] \ &\sim 5 imes 10^{-9} (GeV)^{-2} \left(rac{\lambda_{\Phi H}}{0.5}
ight)^2 \left(rac{1 \mathrm{TeV}}{M_
ho}
ight)^2. \end{aligned}$$

The channels $\rho \rho \rightarrow SS, \omega \omega$ open only when $M_{\rho} > 2$ TeV.



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Indirect search at LHC (heavy S)

A universal cos² θ suppression to all signal strengthes due to H - S mixing. M_S > 40GeV detectable at LHC14 with 3ab⁻¹.
the SM Higgs triple coupling λSM_{HHH} = 3M²_H/v_H is modified to λ_{HHH} = 6λ_Hv_Hc³_θ - 6λ_Sv_Ss³_θ + 3λ_{SH}s_θc_θ(v_Hs_θ - v_Sc_θ). And δ_{HHH} ≡ (λ_{HHH} - λSM_{HHH})/λSM_{HHH}
similarly, λSM_{4H} = 6λ_H = 3(M_H/v_H)² is replaced by λ_{4H} = 6(λ_Hc⁴_θ + λ_Ss⁴_θ) in this model. δ_{4H} ≡ (λ_{4H} - λSM_{4H})/λSM_{4H}, XS for triple Higgs production is too small.



Minimal Model of Majoronic Dark Radiation and Dark Matter an

S decay

The relevant modes are s into quarks, leptons, and ω 's.

$$\Gamma(s \to \omega \omega) = \frac{1}{32\pi} \frac{c_{\theta}^2 M_s^3}{v_s^2}, \Gamma(s \to f\bar{f}) = \frac{M_s}{8\pi} N_c^f \beta_f^3 \left(\frac{m_f s_{\theta}}{v}\right)^2$$

where $eta_f = (1 - rac{4m_f^2}{M_s^2})^{1/2}$. Dominate decay modes: ω -pair $/bar{b}$.



At Z factory (light S)



Future Circular Collider expects to have 10¹²⁻¹³ Z bosons at √s = M_Z with multi-ab⁻¹ luminosity.(JHEP1401,164)
 Defining y_f = M²_{ff}/M²₇ we obtain

$$\frac{dBr(Z \to Sf\bar{f})}{dy} = \frac{g^2 \sin^2 \theta}{192\pi^2 \cos^2 \theta_W} \sqrt{y_f^2 - 2y_f (1 + r_Z^2) + (1 - r_Z^2)^2} \\ \times \frac{\left[y_f^2 + 2y_f (5 - r_Z^2) + (1 - r_Z^2)^2\right]}{(1 - y_f)^2} \times Br(Z \to f\bar{f})$$

where $r_Z = \frac{M_S}{M_Z}$ and $0 \le y_f \le (1 - r_Z)^2$. The kinematic lower bound can be safely taken to be zero even for y_b .



• Note the lower bound for each decay mode.

•
$$Br(Z \rightarrow b\bar{b} \not\in)_{SM} = 5.25 \times 10^{-8}$$

• $Z \rightarrow S\bar{f}f$ signal stands out from the SM background.

DM bound state?



 DM ρ - ρ interact by exchanging t-channel S and H and this force is attractive. The relevant terms are:

$$\mathcal{L} \supset \frac{1}{2} [\lambda_{\Phi H} v_H h + \bar{\kappa} s] \rho^2$$

- $\bar{\kappa} \gg \lambda_{\Phi H} v_H$, s mediation dominates.
- $\bar{\kappa}/M_{\rho} \in [-1.0, 1.0] \Rightarrow \text{DM}$ may form bound state, B_{ρ} .
- Write the effective coupling between $B_{
 ho}$ and 2 ho as

$$\mathcal{L} \sim \alpha_B B_\rho \rho^2$$
.

By dimensional analysis, $\alpha_B \sim (\bar{\kappa}^2/M_{
ho})$.

DM bound state decay Width

- Decay width of B_{ρ} , $\Gamma_B \propto |\psi(0)|^2 \times |\mathcal{M}_{B_{\rho}}|^2$.
- $|\psi(0)|^2 \sim ar\kappa^6/M_
 ho^3$ by dimension analysis.
- Rescale the decay amplitude square and make it dimensionless, broken into

$$|\mathcal{M}_{B_{\rho}}|^{2} = \gamma_{ss} + \gamma_{HH} + \gamma_{sH} + \gamma_{\omega\omega} + \gamma_{W,Z} + \gamma_{f\bar{f}}$$

subscripts label the decay final state.

• drop terms of $\mathcal{O}(v_H/M_{\rho})$.

$$\gamma_{ss} \simeq \left[\lambda_{\Phi S} - \frac{\bar{\kappa}^2}{M_{\rho}^2}
ight]^2, \ \gamma_{HH} \simeq \lambda_{\Phi H}^2,$$

 $\gamma_{\omega\omega} \simeq \left[\lambda_{\Phi S} - \frac{\kappa^2}{M_{\rho}^2 - \kappa v_S}
ight]^2, \ \gamma_{W,Z} \simeq 3\lambda_{\Phi H}^2$

• Finally, the decay width:

$$\Gamma_B \sim M_
ho \left(rac{ar\kappa}{M_
ho}
ight)^6 [\gamma_{ss} + \gamma_{\omega\omega} + 4\lambda_{\Phi H}^2]$$

$\langle \sigma v \rangle$ and DM bound state

• Put Γ_B into the propagator squared, annihilation XS due to B_ρ resonant

$$\sigma \mathbf{v} \sim \frac{\alpha_B^2 (\Gamma_B / M_B)}{(s - M_B^2)^2 + \Gamma_B^2 M_B^2}$$

 Γ_B/M_B takes care the nearly on-shell $B_
ho$ decay.

 $\bullet\,$ When $\, v \ll 1, \, s \sim M_B^2,$ no temperature dependence,

$$\langle \sigma v \rangle \sim \frac{\alpha_B^2}{M_B^3 \Gamma_B} \sim \frac{R_B}{M_\rho^2} [\gamma_{ss} + \gamma_{\omega\omega} + 4\lambda_{\Phi H}^2]$$

and

$$R_B \equiv \left(rac{M_
ho}{ar\kappa}
ight)^2 [\gamma_{ss} + \gamma_{\omega\omega} + 4\lambda_{\Phi H}^2]^{-2}$$

is the boost factor for indirect DM detection.

• For $|\mathcal{M}_{B_{\rho}}|^2 \sim \mathcal{O}(10^0)$ and $\bar{\kappa} \sim 0.1 M_{\rho}$, the boost factor around 100.

At IceCube

- DM annihilate into Majoron pair is a few to 40%. With boost factor 100, $\langle \sigma v \rangle (DM + DM \rightarrow \omega \omega) \sim 10^{-26} 10^{-24} (cm^3/s)$.
- ω could be a component of the 'apparent' neutrino flux at $E_{\nu} = M_{\rho}$ in IceCube and other neutrino observatories.
- Shower events, mostly from the Galactic center.



Summary

- Minimal Majoron model with SM singlet scalars carrying lepton numbers takes care of DR+DM+ m_{ν} +V.S.
- $riangle N_{eff} \sim$ 0.05, or $T_{dec} > m_c$ is preferred.
- Scalar DM, ρ , of mass 1.5 4 TeV is required by V.S. and an operational type-I see-saw.
- $M_S \in [10, 100]$ GeV, mixing as large as 0.1.
- S mainly decays into $b\bar{b}$ and/or $\omega\omega$.
- Sensitive search will be $Z \rightarrow S + f\bar{f}$, followed by S into a pair of Majoron and/or b-quarks.
- Possible DM bound state due to sizable $S\rho\rho$ -coupling.
- shower-like events with apparent neutrino energy at M_{ρ} could contribute to the neutrino flux in underground neutrino detectors.

At IceCube

- IceCube/DeepCore All-flavour Search for Neutrinos from DM Annihilations in the MW(1606.00209)
- Taking $M_{
 ho} = 2$ TeV as example. In our model,

$$\langle \sigma v \rangle_{\omega\omega} = (0 - 0.75) \times 10^{-7} (GeV)^{-2} \times \left(\frac{2\text{TeV}}{M_{
ho}}\right)^2 \times \left(\frac{BF}{100}\right)$$

• IceCube: $< 2 \times 10^{-6} (GeV)^{-2}$ for NFW, and $< 10^{-5} (GeV)^{-2}$ for Burkert,

